SPRING 2021: MATH 147 QUIZ 8

You must show all work to receive full credit. Each problem is worth 5 points.

1. Using spherical coordinates, rewrite, but do not calculate, the triple integral $\int\int\int_B \frac{1}{(x^2+y^2+z^2)^{\frac{3}{2}}}\,dV$, where B is the solid bounded by the spheres $x^2+y^2+z^2=a^2$ and $x^2+y^2+z^2=b^2$, with $0<b<a$.

Solution. Recalling that in spherical coordinates we have $dV=\rho^2\sin(\phi)$ and $x^2+y^2+z^2=\rho^2$, so that $(x^2+y^2+z^2)^{\frac{3}{2}}=\rho^3$, we have

$$\int\int\int_B \frac{1}{(x^2+y^2+z^2)^{\frac{3}{2}}}\,dV = \int_0^{2\pi}\int_0^{\pi}\int_b^a \frac{1}{\rho^3}\cdot\rho^2\sin(\phi)\,d\rho\,d\phi\,d\theta.$$

2. Using a change of variables, set up the triple integral giving the volume of the football B in the shape of the spheroid $\frac{x^2+y^2}{a^2}+\frac{z^2}{b^2}=1$. You do not have to calculate the volume.

Solution. We take $G(u,v,w)=(au,av,bw)$, so that

$$1=\frac{x^2+y^2}{a^2}+\frac{z^2}{b^2}=\frac{(au)^2+(av)^2}{a^2}+\frac{(bw)^2}{b^2}=u^2+v^2+w^2,$$

showing that G takes the sphere B_0 of radius one centered at the origin in the u,v,w coordinate system to B in the x,y,z coordinate system. We also have that

$$\frac{\partial(x,y,z)}{\partial(u,v,w)}=\det\begin{pmatrix}a&0&0\\0&a&0\\0&0&b\end{pmatrix}=a^2b.$$

Since $a>b>0$, $|a^2b|=a^2b$. Thus,

$$\begin{aligned}\text{vol}(B)&=\int\int\int_B dV\\&=\int\int\int_{B_0} a^2b\,dV\\&=\int_0^{2\pi}\int_0^{\pi}\int_0^1 a^2b\cdot\rho^2\sin(\phi)\,d\rho\,d\phi\,d\theta.\end{aligned}$$

3. Suppose $\mathbf{r}(t)=e^t\cdot\boldsymbol{\sigma}(t)$, where $\boldsymbol{\sigma}(t)=e^ti+\sin(t)j+t^3k$. Find $\mathbf{r}'(t)$ in two different ways.

Solution. First, we may rewrite $\mathbf{r}(t)=e^{2t}i+e^t\sin(t)j+t^3e^tk$. Therefore,

$$\mathbf{r}'(t)=2e^{2t}i+(e^t\sin(t)+e^t\cos(t))j+(3t^2e^t+t^3e^t)k.$$

Second, we use the product rule

$$\begin{aligned}\mathbf{r}'(t)&=(e^t)'\boldsymbol{\sigma}(t)+e^t\boldsymbol{\sigma}'(t)\\&=e^t(e^ti+\sin(t)j+t^3k)+e^t(e^ti+\cos(t)j+3t^2k)\\&=2e^{2t}i+(e^t\sin(t)+e^t\cos(t))j+(t^3e^t+3t^2e^t)k.\end{aligned}$$